# Well-posedness of Complex Fluids

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**Abstract:**

Complex fluids are prevalent in many important physical, biological and engineering applications. This talk will address some of the fundamental analytical issues of viscoelastic materials of complex fluids. More precisely, we will present the global well-posedness of classical solutions for (1) and (2) with small initial data, and (3) and (4) with general data.

(1) Infinite Weissenberg number, macroscopic system reads (Lin-Liu-Zhang 05 CPAM, Lei-Zhou 05 SIMA, Lei-Liu-Zhou 08 ARMA, Chen-Zhang 06 CPDE)

$$\begin{cases} u_t + u \cdot \nabla u + \nabla p = \nabla \cdot \Big(\frac{\partial W(F)}{\partial F} F^T\Big) + \mu \Delta u, \\ F_t + u \cdot \nabla F = \nabla u F, \quad \nabla \cdot u = 0. \end{cases} \tag{1}$$

Here $u$ is the velocity field, $p$ is the pressure, $F$ is the deformation tensor and $\tau = \frac{\partial W(F)}{\partial F} F^T$ is the elastic Cauchy stress tensor.

(2) Small strain case, Strain-Rotation model 2D Strain-Rotation Model (Lei 10 ARMA): $\tau = VV^T + 2V$, the dynamics of strain and rotation

$$\begin{cases} V_t + u \cdot \nabla V = D(u) + \frac{1}{2}(\nabla u V + V \nabla u^T) + \frac{1}{2}\big(\omega_{12}(u) - \gamma\big)(VA - AV), \\ \nabla^\perp \theta_t + u \cdot \nabla \nabla^\perp \theta = \frac{1}{2}\Delta u - \Big( - \nabla_2 u \cdot \nabla \theta, \nabla_1 u \cdot \nabla \theta\Big)^T + \nabla^\perp \gamma. \end{cases} \tag{2}$$

Here $A$ is an anti-symmetric constant matrix and $\gamma$ is a nonlinear term.

(3) Micro-Macro FENE model in $R^2$: $\tau = \int_{B(0,R_0)} (R \otimes \nabla_R \mathcal{U}) \psi(\sqcup, \S, \mathcal{R}) \lceil \mathcal{R}$

$$\partial_t \psi + (u \cdot \nabla)\psi = \mathrm{div}_R \big[ - W(u) \cdot R\psi + \beta \nabla_R \psi + \nabla_R \mathcal{U} \psi \big]. \tag{3}$$

Here $\psi(t, x, R)$ is the distribution function and $\mathcal{U}(\mathcal{R}) = -\| \ln(\infty - |\mathcal{R}|^{\in}/|\mathcal{R}_{\prime}|^{\in})$ is the spring potential. The BC: $(\nabla_R \mathcal{U}\psi + \beta \nabla_{\mathcal{R}} \psi) \cdot \mathbf{n} = \prime, \quad$ on $\partial \mathcal{B}(\prime, \mathcal{R}_{\prime})$.

(4) Micro-Macro Smoluchowski equation coupled with NS:

$$\partial_t \psi + (u \cdot \nabla)\psi + \mathrm{div}_g (G(u, \psi)\psi) - \Delta_g \psi = 0, \quad (x, m) \in R^2 \times M. \tag{4}$$

Here $M$ is a $n$-D smooth compact Riemannian manifold without boundary, $G(u, \psi) = \nabla_g \mathcal{U} + \rfloor^{\rangle|}_{\alpha}(\Updownarrow)\partial_{|}\sqcap_{\rangle}$ stands for a meanfield potential with $\mathcal{U}(\sqcup, \S, \Updownarrow) = \int_{\mathcal{M}} \mathcal{K}(\Updownarrow, \amalg)\psi(\sqcup, \S, \amalg)\lceil\amalg$ and the stress tensor is given by

$$\tau_{ij}(t, x) = \int_M \gamma^{(1)}_{ij}(m)\psi(t, x, m) dm + \int_M \int_M \gamma^{(2)}_{ij}(m_1, m_2)\psi(t, x, m_1)\psi(t, x, m_2) dm_1 dm_2.$$